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# Spontaneous CP violation in non-minimal supersymmetric models

**Cyril Hugonie**

*AHEP Group, Instituto de Física Corpuscular – CSIC/Universitat de València*
*Edificio Institutos de Investigación*
*Apartado de Correos 22085, E-46071 València, Spain*
*E-mail:* c.hugonie@ific.uv.es

**Jorge C. Romão**[a] **and Ana M. Teixeira**[ab]

[a]*Departamento de Física and CFIF, Instituto Superior Técnico*
*Av. Rovisco Pais, P-1049-001 Lisboa, Portugal*
[b]*Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI*
*Universidad Autónoma de Madrid*
*Cantoblanco, E-28049 Madrid, Spain*
*E-mail:* jorge.romao@ist.utl.pt, ana@cfif.ist.utl.pt

Abstract: We study the possibilities of spontaneous CP violation in the Next-to-Minimal Supersymmetric Standard Model with an extra singlet tadpole term in the scalar potential. We calculate the Higgs boson masses and couplings with radiative corrections including dominant two loop terms. We show that it is possible to satisfy the LEP constraints on the Higgs boson spectrum with non-trivial spontaneous CP violating phases. We also show that these phases could account for the observed value of $\varepsilon_K$.

Keywords: Supersymmetry Phenomenology, Beyond Standard Model, CP violation.

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## 1. Introduction

The understanding of CP violation, first observed in $K$ decays [1], remains an open and most challenging question in particle physics. In the Standard Model (SM), CP violation arises from the presence of complex Yukawa couplings in the lagrangian. In electroweak interactions, CP violation originates in the misalignment of mass and charged-current interaction eigenstates, and is parameterized by the physical phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [2], $\delta_{CKM}$. Although SM predictions are in good agreement with experimental observations, including recent measurements at $B$-factories, the SM amount of CP violation fails to account for the observed baryon asymmetry of the Universe [3]. Moreover, one is yet to find an answer for the strong CP problem, or in other words, to understand the smallness of the $\bar{\theta}$ parameter. Bounds from the electric dipole moment (EDM) of the neutron force this flavour-conserving CP violating phase to be as small as $10^{-10}$, and this fine-tuning is most unnatural in the sense of 't Hooft [4], since the lagrangian does not acquire any new symmetry in the limit where $\bar{\theta}$ vanishes.

In supersymmetric (SUSY) extensions of the SM there are additional sources of explicit CP violation, arising from complex soft SUSY breaking terms as well as from the complex SUSY conserving $\mu$ parameter. As pointed in [5], these phases can account for the values of the $K$ and $B$ meson CP violating observables, even in the absence of $\delta_{CKM}$. However, the supersymmetric phases also generate large contributions to the EDMs of the electron, neutron and mercury atom. The non-observation of the EDMs imposes strong constraints on the SUSY phases, forcing them to be very small. Putting these new phases to zero is also not natural in the sense of 't Hooft. This is the so-called SUSY CP problem and many are the solutions that have been proposed to overcome it (for a review see refs. [6]).

An attractive approach to the SUSY CP problem is to impose CP invariance on the lagrangian, and spontaneously break it through complex vacuum expectation values (VEVs) for the Higgs scalar fields [7]. Thus, CP symmetry is restored at high energy and CP violating phases appear as dynamical variables. Spontaneous CP violation (SCPV) is also an appealing solution to the strong CP problem, since in this case one naturally has a vanishing $\bar{\theta}$ at tree level [8]. Further motivation to SCPV stems from string theories, where it has been shown that in string perturbation theory CP exists as a good symmetry that could be spontaneously broken [9].

SCPV requires at least two Higgs doublets. The Minimal Supersymmetric Standard Model (MSSM) is a very appealing example of a two Higgs doublet model. However, it is well known that, at tree level, SCPV does not occur in the MSSM [10]. On the other hand, radiative corrections can generate CP violating operators [11] but then, according to the Georgi-Pais theorem on radiatively broken global symmetries [12], one expects to have light Higgs states in the spectrum [13], which are excluded by LEP [14]–[16].

The case of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [17, 18], where a singlet superfield is added to the Higgs sector, is more involved. In the usual $\mathbb{Z}_3$ invariant version of the model, where only dimensionless couplings are allowed in the superpotential, it has been shown that, although CP violating extrema are present at tree level, these are always maxima, not minima of the scalar potential, with negative squared masses for the Higgs states [19]. Hence SCPV is not feasible at tree level in the NMSSM. Furthermore, as in the MSSM case, radiatively induced CP violating minima always bear light Higgs states which are difficult to accommodate with LEP data [20, 21]. The possibility of SCPV has been studied in more general non-minimal models, where no $\mathbb{Z}_3$ symmetry is assumed and dimensionful, SUSY conserving terms are present in the superpotential [22]–[24]. In this case, it has been shown that SCPV is possible and could account for the observed value of $\varepsilon_K$ in the kaon system [22, 24].

The first drawback of the general NMSSM with respect to the $\mathbb{Z}_3$ invariant version, is that it no longer provides a solution to the $\mu$ problem of the MSSM, which was one of the original motivations of the NMSSM. The second is that, in the absence of a global symmetry under which the singlet field is charged, divergent singlet tadpoles proportional to $M_{\text{Planck}}$, generated by non-renormalizable higher order interactions, can appear in the effective scalar potential [25, 26]. Such tadpole terms would destabilize the hierarchy between the electroweak (EW) scale and the Planck scale. On the other hand, if $\mathbb{Z}_3$, or any other discrete symmetry, is imposed at the lagrangian level, it is spontaneously broken at the EW scale once the Higgs fields get non-vanishing VEVs, giving rise to disastrous cosmological domain walls [27]. It has recently been argued that using global discrete $R$-symmetries for the complete theory - including non-renormalizable interactions - one could construct a $\mathbb{Z}_3$ invariant renormalizable superpotential and generate a $\mathbb{Z}_3$ breaking non-divergent singlet tadpole term in the scalar potential [28, 29]. These models are free of both stability and domain wall problems, and all the dimensionful parameters, including the singlet tadpole, are generated through the soft SUSY breaking terms.

The aim of this paper is to study the possibility of SCPV in the NMSSM with an extra singlet tadpole term in the effective potential, taking into account the latest experimental constraints on Higgs boson and sparticle masses, as well as the observed value of $\varepsilon_K$. In particular, contrary to what was asserted in ref. [30], we obtain that in the $\mathbb{Z}_3$ symmetric limit, SCPV *cannot* be accommodated with the LEP exclusion limit on a light Higgs boson. On the other hand, we show that if $\mathbb{Z}_3$ is broken by a non-zero singlet tadpole, it is possible to spontaneously break CP, satisfy the LEP constraints on the Higgs boson mass and have $\varepsilon_K$ compatible with the experimental value, although typically favouring a large phase solution. The paper is organized as follows: in section 2 we define the model and derive the Higgs boson mass matrix. The procedure to scan the parameter space and the resulting Higgs boson spectrum are discussed in section 3. Section 4 is devoted to the calculation of $\varepsilon_K$. Finally, we present the conclusions in section 5.

## 2. Overview of the model

### 2.1 Higgs scalar potential

In addition to the Yukawa couplings for the quarks and leptons (as in the MSSM), the superpotential of the NMSSM is defined by

$$W_{\text{Higgs}} = \lambda \hat{H}_1 \hat{H}_2 \hat{N} + \frac{1}{3}\kappa \hat{N}^3 , \tag{2.1}$$

where $\hat{H}_1 = (\hat{H}_1^0, \hat{H}_1^-)$ is the Higgs doublet superfield coupled to the down-type fermions, $\hat{H}_2 = (\hat{H}_2^+, \hat{H}_2^0)$ the one coupled to the up-type ones, and $\hat{N}$ is a singlet. Once EW symmetry is broken, the scalar component of $\hat{N}$ acquires a VEV, $x = |\langle N \rangle|$, thus generating an effective $\mu$ term

$$\mu \equiv \lambda x . \tag{2.2}$$

The superpotential in eq. (2.1) is scale invariant, and the EW scale appears only through the soft SUSY breaking terms. It is also invariant under a global $\mathbb{Z}_3$ symmetry. The possible domain wall problem due to the spontaneous breaking of the $\mathbb{Z}_3$ symmetry at the EW scale is assumed to be solved by adding non-renormalizable interactions which break the $\mathbb{Z}_3$ symmetry without spoiling the quantum stability with unwanted divergent singlet tadpoles. This can be achieved by replacing the $\mathbb{Z}_3$ symmetry by a set of discrete $R$-symmetries, broken by the soft SUSY breaking terms [17, 28]. At low energy, the additional non-renormalizable terms allowed by the $R$-symmetries generate an extra linear term for the singlet in the effective potential, through tadpole loop diagrams

$$V_{\text{tadpole}} = -\xi^3 N + \text{h.c.} , \tag{2.3}$$

where $\xi$ is of the order of the soft SUSY breaking terms ($\lesssim$ 1 TeV). Since our approach is phenomenological, we take $\xi$ as a free parameter, without considering the details of the non-renormalizable interactions that generate it. Likewise, we do not discard the singlet self-coupling term in eq. (2.1) by imposing $\kappa = 0$, which is possible once $\xi \neq 0$ [29], but rather assume $\kappa$ to be a free parameter.

In addition to $V_{\rm tadpole}$, the tree level Higgs potential has the usual $F$ and $D$ terms as well as soft SUSY breaking terms:

$$
\begin{aligned}
V_F &= \lambda^2\left(|H_1|^2|N|^2+|H_2|^2|N|^2+|H_1|^2|H_2|^2\right)-\lambda^2\left(H_1^{0*}H_2^{0*}H_1^-H_2^+ + \text{h.c.}\right)+\kappa^2|N|^4\,,\\
V_D &= \frac{g_1^2+g_2^2}{8}\left(|H_1|^2-|H_2|^2\right)^2+\frac{g_2^2}{2}|H_1^\dagger H_2|^2\,,\\
V_{\rm soft} &= m_{H_1}^2|H_1|^2+m_{H_2}^2|H_2|^2+m_N^2|N|^2+\left(\lambda A_\lambda N H_1 H_2+\frac{1}{3}\kappa A_\kappa N^3+\text{h.c.}\right), \qquad (2.4)
\end{aligned}
$$

where $g_1$ and $g_2$ are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ coupling constants, respectively. In what follows, the soft SUSY breaking terms $m_{H_1}, m_{H_2}, m_N, A_\lambda, A_\kappa$ are taken as free parameters at the weak scale and no assumption is made on their value at the GUT scale. We assume that the lagrangian is CP invariant, which means that all the parameters appearing in eqs. (2.3) and (2.4) are real. On the other hand, once the EW symmetry is spontaneously broken, the neutral Higgs fields acquire complex VEVs that spontaneously break CP. By gauge invariance, one can take $\langle H_1^-\rangle = 0$. The condition for a local minimum with $\langle H_2^+\rangle = 0$ is equivalent to a positive square mass for the charged Higgs boson. The VEVs of the neutral Higgs fields have the general form

$$
\langle H_1^0\rangle = v_1 e^{i\varphi_1}\,, \qquad \langle H_2^0\rangle = v_2 e^{i\varphi_2}\,, \qquad \langle N\rangle = x e^{i\varphi_3}\,, \qquad (2.5)
$$

where $v_1, v_2, x$ are positive and $\varphi_1, \varphi_2, \varphi_3$ are CP violating phases. However, only two of these phases are physical. They can be chosen as

$$
\theta = \varphi_1+\varphi_2+\varphi_3 \qquad \text{and} \qquad \delta = 3\varphi_3\,. \qquad (2.6)
$$

### 2.2 Minimization of the tree level potential

From the tree level scalar potential in eqs. (2.3) and (2.4), one can derive the five minimization equations for the VEVs and phases $v_1, v_2, x, \theta, \delta$. They can be used to express the soft parameters $m_{H_1}, m_{H_2}$, $m_N, A_\lambda, A_\kappa$ in terms of $v_1, v_2, x, \theta, \delta$:

$$
\begin{aligned}
\frac{\partial V_{\rm tree}}{\partial v_1}=0 \quad &\Rightarrow \quad m_{H_1}^2 = -\lambda^2\left(x^2+v^2\sin^2\beta\right)-\frac{1}{2}M_Z^2\cos 2\beta-\\
&\qquad\qquad -\lambda x\tan\beta\left(\kappa x\cos(\theta-\delta)+A_\lambda\cos\theta\right)\,,\\
\frac{\partial V_{\rm tree}}{\partial v_2}=0 \quad &\Rightarrow \quad m_{H_2}^2 = -\lambda^2\left(x^2+v^2\cos^2\beta\right)+\frac{1}{2}M_Z^2\cos 2\beta-\\
&\qquad\qquad -\lambda x\cot\beta\left(\kappa x\cos(\theta-\delta)+A_\lambda\cos\theta\right)\,,\\
\frac{\partial V_{\rm tree}}{\partial x}=0 \quad &\Rightarrow \quad m_N^2 = -\lambda^2 v^2-2\kappa^2x^2-\lambda\kappa v^2\sin 2\beta\cos(\theta-\delta)-\\
&\qquad\qquad -\frac{\lambda A_\lambda v^2}{2x}\sin 2\beta\cos\theta-\kappa A_\kappa x\cos\delta+\frac{\xi^3}{x}\cos\left(\frac{\delta}{3}\right),\\
\frac{\partial V_{\rm tree}}{\partial \theta}=0 \quad &\Rightarrow \quad A_\lambda = -\frac{\kappa x\sin(\theta-\delta)}{\sin\theta}\,,\\
\frac{\partial V_{\rm tree}}{\partial \delta}=0 \quad &\Rightarrow \quad A_\kappa = \frac{3\lambda\kappa x v^2\sin 2\beta\sin(\theta-\delta)+2\xi^3\sin(\delta/3)}{2\kappa x^2\sin\delta}\,, \qquad (2.7)
\end{aligned}
$$

with $\tan\beta = v_2/v_1$, $v = \sqrt{v_1^2+v_2^2} = 174\,\text{GeV}$ and $M_Z$ the $Z$ boson mass. The above relations allow us to use $\tan\beta, x, \theta$ and $\delta$ instead of $m_{H_1}, m_{H_2}, m_N, A_\lambda, A_\kappa$ as free parameters.

Once EW symmetry is spontaneously broken, we are left with five neutral Higgs states and a pair of charged Higgs states. The neutral Higgs fields can be rewritten in terms of CP eigenstates

$$\begin{aligned} H_1^0 &= e^{i\varphi_1}\left\{v_1 + \frac{1}{\sqrt{2}}(S_1 + i\sin\beta P)\right\}, \\ H_2^0 &= e^{i\varphi_2}\left\{v_2 + \frac{1}{\sqrt{2}}(S_2 + i\cos\beta P)\right\}, \\ N &= e^{i\varphi_3}\left\{x + \frac{1}{\sqrt{2}}(X + iY)\right\}, \end{aligned} \tag{2.8}$$

where $S_1, S_2, X$ are the CP-even components, and $P, Y$ are the CP-odd components. Note that we have rotated away the CP-odd would-be Goldstone boson associated with the EW symmetry breaking. The mass matrix for the neutral Higgs bosons in the basis $(S_1, S_2, X, P, Y)$ can be easily obtained:

$$\begin{aligned} M_{11}^2 &= M_Z^2\cos^2\beta - \frac{\lambda\kappa x^2\tan\beta\sin\delta}{\sin\theta} \\ M_{22}^2 &= M_Z^2\sin^2\beta - \frac{\lambda\kappa x^2\cot\beta\sin\delta}{\sin\theta} \\ M_{33}^2 &= 4\kappa^2x^2 + \frac{1}{2}\lambda\kappa v^2\sin 2\beta\sin(\theta-\delta)\left(\cot\theta + 3\cot\delta\right) + \frac{\xi^3}{x}\left(\sin\left(\frac{\delta}{3}\right)\cot\delta + \cos\left(\frac{\delta}{3}\right)\right) \\ M_{44}^2 &= -\frac{2\lambda\kappa x^2\sin\delta}{\sin 2\beta\sin\theta} \\ M_{55}^2 &= \frac{1}{2}\lambda\kappa v^2\sin 2\beta\left(6\cos(\theta-\delta) - 9\frac{\sin\theta}{\sin\delta} - \frac{\sin\delta}{\sin\theta}\right) + \frac{\xi^3}{x}\left(\cos\left(\frac{\delta}{3}\right) - 3\sin\left(\frac{\delta}{3}\right)\cot\delta\right) \\ M_{12}^2 &= \left(\lambda^2v^2 - \frac{M_Z^2}{2}\right)\sin 2\beta + \frac{\lambda\kappa x^2\sin\delta}{\sin\theta} \\ M_{13}^2 &= 2\lambda^2 xv\cos\beta + \lambda\kappa xv\sin\beta\left(\cos(\theta-\delta) + \frac{\sin\delta}{\sin\theta}\right) \\ M_{14}^2 &= 0 \\ M_{15}^2 &= 3\lambda\kappa xv\sin\beta\sin(\theta-\delta) \\ M_{23}^2 &= 2\lambda^2 xv\sin\beta + \lambda\kappa xv\cos\beta\left(\cos(\theta-\delta) + \frac{\sin\delta}{\sin\theta}\right) \\ M_{24}^2 &= 0 \\ M_{25}^2 &= 3\lambda\kappa xv\cos\beta\sin(\theta-\delta) \\ M_{34}^2 &= -\lambda\kappa xv\sin(\theta-\delta) \\ M_{35}^2 &= -2\lambda\kappa v^2\sin 2\beta\sin(\theta-\delta) \\ M_{45}^2 &= \lambda\kappa xv\left(\cos(\theta-\delta) + \frac{\sin\delta}{\sin\theta}\right), \end{aligned} \tag{2.9}$$

where we made use of the minimization conditions in eq. (2.7) to eliminate the soft terms and simplify the expressions. One can note that $M_{14}^2 = M_{24}^2 = 0$, which means that there is *no CP violation in the Higgs doublet sector*. On the other hand, CP violating mixings between the singlet and the doublets can appear, as long as $\theta \neq \delta$. It is easy to see from eq. (2.9) that, if $\xi \neq 0$, one can evade the NMSSM no-go theorem for SCPV: it was shown in [19] that the tree level mass matrix for the neutral Higgs bosons always had one negative eigenvalue in the case of non-trivial phases of the VEVs. However, in our case the negative eigenvalue can be lifted up to a positive value provided $\xi$ is large enough, due to the additional diagonal terms proportional to $\xi^3$ in $M_{33}^2$ and $M_{55}^2$. Hence, SCPV is possible already *at tree level* for $\xi \neq 0$. We will check this numerically in section 3, taking into account radiative corrections as well as experimental constraints on the Higgs boson masses.

### 2.3 Radiative corrections

It is well known that one loop radiative corrections can give large contributions to the Higgs boson masses in the MSSM [31] as well as in the NMSSM [32]. Furthermore, they play a crucial role in the SCPV mechanism [11, 20], as they generate CP violating operators. In what follows, we shall only consider radiative corrections due to top-stop loops. The one loop effective potential reads

$$V_{1-\text{loop}} = \frac{3}{32\pi^2}\left\{\mathcal{M}_{\tilde{t}_i}^4\left(\log\frac{\mathcal{M}_{\tilde{t}_i}^2}{Q^2} - \frac{3}{2}\right) - 2\mathcal{M}_t^4\left(\log\frac{\mathcal{M}_t^2}{Q^2} - \frac{3}{2}\right)\right\}. \tag{2.10}$$

$\mathcal{M}_t^2$ and $\mathcal{M}_{\tilde{t}_i}^2, (i = 1, 2)$ are the field dependent top and stop squared masses respectively, and $Q$ is the renormalization scale, at which all the parameters are evaluated. $Q$ has to be of the order of the soft SUSY breaking parameters so that the tree level scalar potential has the supersymmetric form of eq. (2.4). The field dependent stop mass matrix, in the basis $(T_L, T_R^{c*})$, is given by

$$\mathcal{M}_{\tilde{t}}^2 = \begin{bmatrix} m_Q^2 + h_t^2|H_2^0|^2 & h_t(A_tH_2^{0*} + \lambda H_1^0 N) \\ h_t(A_tH_2^0 + \lambda H_1^{0*}N^*) & m_T^2 + h_t^2|H_2^0|^2 \end{bmatrix}, \tag{2.11}$$

with $h_t$ the top Yukawa coupling and $m_Q, m_T, A_t$ the soft terms for the stop sector. In the following, we assume $m_Q = m_T \equiv M_{\text{SUSY}}$, as this choice maximizes the radiative corrections to the lightest Higgs boson mass. $D$ terms are not taken into account since we do not consider radiative corrections proportional to the gauge couplings. At the minimum of the potential, the top-stop masses are given by

$$\begin{aligned} m_t^2 &= h_t^2 v^2 \sin^2\beta\,, \\ m_{\tilde{t}_1,\tilde{t}_2}^2 &= M_{\text{SUSY}} + m_t^2 \mp m_t X_t\,, \end{aligned} \tag{2.12}$$

where

$$X_t = \sqrt{A_t^2 + \lambda^2 x^2 \cot^2\beta + 2A_t\lambda x\cot\beta\cos\theta} \tag{2.13}$$

is the usual stop mixing parameter. This is not different from the CP conserving case, up to the phase $\theta$ appearing in $X_t$. One can show that radiative corrections to the Higgs boson masses are minimized for $X_t = 0$ (minimal mixing scenario) and maximized for

$X_t = \sqrt{6} M_{\rm SUSY}$ (maximal mixing scenario). However, in our case it is not possible to take $X_t = 0$. In fact, the minimum of $X_t$ is $X_t = \lambda x \cot\beta |\sin\theta|$ for $A_t = -\lambda x \cot\beta \cos\theta$. On the other hand, $X_t = \sqrt{6} M_{\rm SUSY}$ is only possible if $\lambda x \cot\beta |\sin\theta| < \sqrt{6} M_{\rm SUSY}$. If this is not the case, then the maximal mixing scenario is obtained for $X_t = \lambda x \cot\beta |\sin\theta|$ and $A_t = -\lambda x \cot\beta \cos\theta$ as in the minimum mixing case.

The one loop terms give additional contributions to the minimization conditions of eq. (2.7). In particular, since $m^2_{\tilde{t}_i}$ depends on the CP violating phase $\theta$, the relation that gives $A_\lambda$ as a function of the VEVs and phases of the Higgs fields is no longer valid. Moreover, as noted before, the tree level minimization equations were used to simplify the Higgs boson mass matrix elements at tree level. Nevertheless, it is possible to keep the tree level mass matrix elements as in eq. (2.9) and write all the one loop contributions as additional terms in the Higgs boson mass matrix

$$M^2_{ij} \to M^2_{ij} + \frac{3 h_t^2 m_t^2}{8\pi^2}\, \delta M^2_{ij} \,. \tag{2.14}$$

One then obtains

$$\begin{aligned}
\delta M^2_{11} &= -\lambda^2 x^2 \Delta_1^2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{22} &= -A_t^2 \Delta_2^2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + 2 A_t \Delta_2\, g(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + \log \frac{m^2_{\tilde{t}_1} m^2_{\tilde{t}_2}}{m_t^4} \\
\delta M^2_{33} &= -\lambda^2 v^2 \cos^2\beta \Delta_1^2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{44} &= -\frac{\lambda^2 x^2 A_t^2 \sin^2\theta}{\sin^2\beta}\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{55} &= -\lambda^2 v^2 \cos^2\beta A_t^2 \sin^2\theta\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{12} &= -\lambda x A_t \Delta_1 \Delta_2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + \lambda x \Delta_1\, g(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{13} &= -\lambda^2 x v \cos\beta \Delta_1^2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + \frac{\lambda^2 x v \cos\beta}{m_t^2}\, h(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{14} &= \frac{\lambda^2 x^2 A_t \sin\theta \Delta_1}{\sin\beta}\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{15} &= \lambda^2 x v \cos\beta A_t \sin\theta \Delta_1\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{23} &= -\lambda v \cos\beta A_t \Delta_1 \Delta_2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + \lambda v \cos\beta \Delta_1\, g(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{24} &= \frac{\lambda x A_t^2 \sin\theta \Delta_2}{\sin\beta}\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) - \frac{\lambda x A_t \sin\theta}{\sin\beta}\, g(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{25} &= \lambda v \cos\beta A_t^2 \sin\theta \Delta_2\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) - \lambda v \cos\beta A_t \sin\theta\, g(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{34} &= \frac{\lambda^2 x v \cos\beta A_t \sin\theta \Delta_1}{\sin\beta}\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{35} &= \lambda^2 v^2 \cos^2\beta A_t \sin\theta \Delta_1\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
\delta M^2_{45} &= -\frac{\lambda^2 x v \cos\beta A_t^2 \sin^2\theta}{\sin\beta}\, f(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \,,
\end{aligned} \tag{2.15}$$



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where

$$f(m_1,m_2) = \frac{1}{(m_2^2-m_1^2)^2}\left(\frac{m_1^2+m_2^2}{m_2^2-m_1^2}\log\frac{m_2^2}{m_1^2}-2\right),$$
$$g(m_1,m_2) = \frac{1}{m_2^2-m_1^2}\log\frac{m_2^2}{m_1^2},$$
$$h(m_1,m_2) = \frac{1}{m_2^2-m_1^2}\left[m_2^2\log\frac{m_2^2}{Q^2}-m_1^2\log\frac{m_1^2}{Q^2}\right]-1, \tag{2.16}$$

and

$$\Delta_1 = A_t\cos\theta+\lambda x\cot\beta\,,\qquad \Delta_2 = A_t\cos\theta+\lambda x\cot\beta\cos\theta\,. \tag{2.17}$$

One can notice that, apart from a sub-leading term in $\delta M_{13}^2$, all the scale dependence is hidden in the parameters scale dependence. The one loop terms of the neutral Higgs boson mass matrix in eq. (2.15) differ from those given in ref. [30, eq. (9)]. In the latter, the authors computed the one loop effective potential for the top/stop sector including the contributions of the $D$-terms in the stop mass matrix (which we neglect), from which they deduced the one loop minimization equations and the one loop Higgs boson mass matrix. However, in order to simplify the tree level Higgs boson mass matrix, they used tree level minimization conditions (which, in particular, leads to $M_{14}=M_{24}=0$ in our eq. (2.9)), without taking into account the one loop effects. We believe this is the reason why the authors of ref. [30] claimed it is possible to have a rather heavy Higgs spectrum in the $\mathbb{Z}_3$ invariant NMSSM with spontaneous CP violation, in contradiction with what has been previously obtained in the literature [20, 21], and with what one expects from the Georgi-Pais theorem [12] as well as from the Romão no-go theorem at tree level [19]. As we will see in section 3, we also found it difficult, if not impossible, to have spontaneous CP violation in the $\mathbb{Z}_3$ invariant limit where $\xi\to 0$.

Two loop corrections can give substantial contributions to the Higgs boson masses [33, 34]. Our analysis follows closely the results of [34], to which we refer the reader for more details. Here, we consider the dominant two loop corrections which are proportional to $\alpha_s h_t^4$ and $h_t^6$, taking only the leading logarithms (LLs) into account. In this approximation, the two loop effective potential reads

$$V_{2-\mathrm{loops}}^{\mathrm{LL}} = 3\left(\frac{h_t^2}{16\pi^2}\right)^2\left(32\pi\alpha_s-\frac{3}{2}h_t^2\right)v^4\sin^4\beta\,\log^2\frac{m_t^2}{Q^2}\,. \tag{2.18}$$

One loop corrections to the tree level relations between bare parameters and physical observables, once reinserted in the one loop potential of eq. (2.10), also appear as two loop effects. The dominant contributions are:

(i) corrections to the kinetic terms of the Higgs bosons, which lead to a wave function renormalization factor $Z_{H_2^0}$ given by

$$Z_{H_2^0} = 1-3\frac{h_t^2}{16\pi^2}\log\frac{m_t^2}{Q^2}\,; \tag{2.19}$$

(ii) corrections to the top quark Yukawa coupling

$$h_t(m_t) = h_t(Q)\left(1 + \frac{1}{32\pi^2}\left(\frac{9}{2}\,h_t^2 - 32\pi\alpha_s\right)\,\log\frac{m_t^2}{Q^2}\right). \tag{2.20}$$

The top quark running mass is then given by $m_t(m_t) = h_t(m_t) Z_{H_2^0}^{1/2} v \sin\beta$ and the relation between the pole and running masses is, up to order $\alpha_s$, $m_t^{\rm pole} = (1+\frac{4\alpha_s}{3\pi}) m_t(m_t)$. Similarly, the relation between the stop pole and running masses is given by $m_{\tilde{t}_i}^{\rm pole} = (1+\frac{8\alpha_s}{3\pi}) m_{\tilde{t}_i}(m_{\tilde{t}_i})$.

Once all these contributions are taken into account, one obtains a rather complicated $5\times 5$ mass matrix for the neutral Higgs fields, which can only be numerically diagonalized (cf. section 3).

### 2.4 Charged Higgs boson mass

Finally, the expression for the charged Higgs boson mass at tree level is given by

$$m_{H^\pm}^2 = M_W^2 - \lambda^2 v^2 - \frac{2\lambda\kappa x^2 \sin\delta}{\sin 2\beta \sin\theta}\,. \tag{2.21}$$

The radiative corrections due to top-bottom and stop-sbottom loops read

$$\delta m_{H^\pm}^2 = -\frac{3h_t^2}{8\pi^2}\frac{\lambda x A_t \cos\theta}{\sin 2\beta}\; h(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + \delta_r\,, \tag{2.22}$$

where $h(m_1, m_2)$ has been defined in eq. (2.16) and $\delta_r$ contains the scale independent terms

$$\begin{aligned}\delta_r &= \frac{3h_t^2 m_t^2}{32\pi^2}\frac{\lambda^2 x^2}{\sin^2\beta}\Bigg[\frac{m_{\tilde{b}_L}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{b}_L}^2)(m_{\tilde{t}_2}^2 - m_{\tilde{b}_L}^2)}\;\log\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_{\tilde{b}_L}^4} - \\ &\qquad - \frac{1}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{b}_L}^2} + \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2 - m_{\tilde{b}_L}^2}\right)\;\log\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\Bigg] \\ &\simeq -\frac{3h_t^2}{16\pi^2}\frac{\lambda^2 x^2 v^2}{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}\,. \end{aligned} \tag{2.23}$$

In the above, we have assumed that the bottom Yukawa coupling is small enough to be neglected, as justified by our choice of parameters, namely a low $\tan\beta$ regime. Therefore bottom squarks $\tilde{b}_L$ and $\tilde{b}_R$ do not mix and $m_{\tilde{b}_L} = m_Q \equiv M_{\rm SUSY}$. It is interesting to note that the term proportional to $h(m_{\tilde{t}_1}, m_{\tilde{t}_2})$ in eq. (2.22) exactly cancels the one loop correction to $A_\lambda$ due to the minimization equation of $V_{1-{\rm loop}}$ as a function of $\theta$. Hence, if one replaces $A_\lambda$ by a function of the VEVs and phases of the Higgs fields, one can simply rewrite the charged Higgs boson mass as

$$m_{H^\pm}^2 \simeq M_W^2 - \lambda^2 v^2 - \frac{2\lambda\kappa x^2 \sin\delta}{\sin 2\beta \sin\theta} - \frac{3h_t^2}{16\pi^2}\frac{\lambda^2 x^2 v^2}{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}\,. \tag{2.24}$$

## 3. Mass spectrum

In this section we investigate whether it is possible to have SCPV in the NMSSM with the extra tadpole term for the singlet, given the exclusion limits on the Higgs boson spectrum from LEP [14]–[16]. In order to do so, we perform a numerical scanning of the parameter space of the model. The parameters appearing in the tree level Higgs boson mass matrix are $\lambda, \kappa, A_\lambda, A_k, m_{H_1}, m_{H_2}, m_N, \xi$. As seen in the previous section, we can use the minimization conditions of eq. (2.7) to replace the soft SUSY breaking terms by the VEVs and phases of the Higgs fields. In the following, we will use the effective $\mu$ term defined in eq. (2.2) as a free parameter instead of the singlet VEV $x$, so that the free parameters of the tree level Higgs boson mass matrix are now given by

$$\lambda\,,\ \kappa\,,\ \tan\beta\,,\ \mu\,,\ \xi\,,\ \theta\,,\ \delta\,. \tag{3.1}$$

Requiring the absence of Landau singularities for $\lambda$ and $\kappa$ below the GUT scale ($M_{GUT} \sim 2\times 10^{16}$ GeV) imposes upper bounds on these couplings at the weak scale, which depend on the value of $h_t$, i.e. of $\tan\beta$ and the top mass [18]. For $\tan\beta \le 10$ and $m_t^{\rm pole} = 175\,\mathrm{GeV}$, one finds $\lambda_{\max} \sim 0.65$ and $\kappa_{\max} \sim 0.6$. This also yields a lower bound for $\tan\beta$, namely $\tan\beta \gtrsim 2.2$. Regarding the radiative corrections, we take $M_{\rm SUSY} = 350\,\mathrm{GeV}$ and assume the maximal mixing scenario for the stops, i.e. $X_t = \max(\sqrt{6}M_{\rm SUSY}, \mu\cot\beta|\sin\theta|)$. The relatively small value for $M_{\rm SUSY}$ will be justified in the following section when we address the computation of $\varepsilon_K$.

We have performed numerical scannings on the free parameters, which were randomly chosen in the following intervals:

$$\begin{array}{ccc} 0.01 < \lambda < 0.65\,, & 0.01 < \kappa < 0.6\,, & 2.2 < \tan\beta < 10\,, \\ 100\,\mathrm{GeV} < \mu < 500\,\mathrm{GeV}\,, & 0 < \xi < 1\,\mathrm{TeV}\,, & \\ -\pi < \theta < \pi\,, & -\pi < \delta < \pi\,. & \end{array} \tag{3.2}$$

For each point, we computed the Higgs boson masses and couplings by diagonalizing the $5\times 5$ Higgs boson mass matrix, which was calculated taking into account radiative corrections up to the dominant two loop terms, as described in the previous section. The five mass eigenstates are denoted by $h_i, i = 1..5$ with masses $m_i$ in increasing order. We also computed the charged Higgs boson mass, the stop and the chargino masses, and applied all the available experimental constraints on these particles from LEP [14]–[16] and [35, 36], as discussed below.

The first result is that in the $\mathbb{Z}_3$ invariant limit, where $\xi = 0$, we found it extremely difficult, if not impossible, to satisfy the LEP constraints on the Higgs sector with non zero CP violating phases, contrary to what was claimed in ref. [30]. Starting from $\sim 10^9$ points randomly chosen as in eq. (3.2), apart from $\xi$ fixed to 0, we eventually find only a couple of points passing all the experimental tests. These points have in common a very small phase for the singlet VEV, $\delta \sim 0$. Hence, according to eq. (2.24), the charged Higgs boson mass is close to $M_W$, i.e. almost excluded by LEP. This result is easily understood from the fact that in the limit where $\xi \to 0$, SCPV is not possible at tree level [19], and although viable when radiative corrections are included, the Georgi-Pais theorem [12] predicts the appearance



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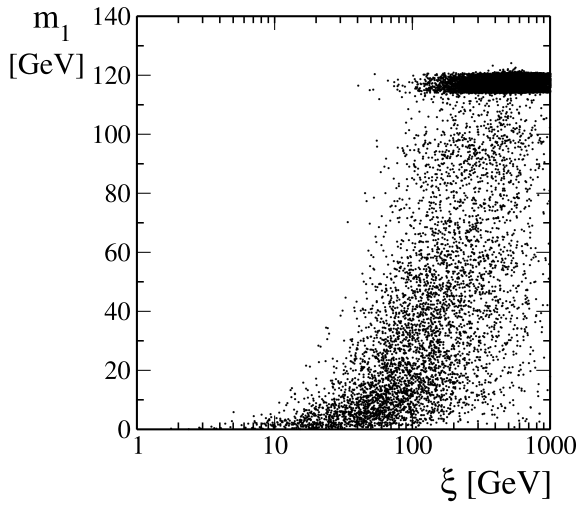


**Figure 1:** Mass of the lightest Higgs state as a function of $\xi$, for a $m_t^{\rm pole} = 175\,{\rm GeV}$, $M_{\rm SUSY} = 350\,{\rm GeV}$ and maximal stop mixing. The other parameters are randomly chosen as in eq. (3.2).

of light Higgs states in the spectrum, already excluded by LEP. A few benchmark points with SCPV in the $\mathbb{Z}_3$ invariant NMSSM were proposed in refs. [20, 21]. These points had a rather light neutral Higgs boson spectrum, now excluded by LEP searches. Moreover, they had $\tan\beta = 1$, which is excluded if one requires no Landau poles for $h_{top}, \lambda$ and $\kappa$ below the GUT scale, as noted above. Finally, the large value of $M_{\rm SUSY}$ for these points ($M_{\rm SUSY} = 3\,{\rm TeV}$) would be difficult to accommodate with the experimental value of $\varepsilon_K$, as we shall see in the next section.

On the other hand, when $\xi \neq 0$, CP can be spontaneously broken already at tree level, as noted in the previous section. Taking into account radiative corrections up to the dominant two loop terms, we obtained that large portions of the parameter space of eq. (3.2) complied with all the imposed constraints for any values of the CP violating phases $\theta$ and $\delta$, i.e. it is possible to have SCPV in the NMSSM.

In figure 1 we display the mass of the lightest Higgs boson, $m_1$, as a function of the tadpole parameter $\xi$, with the other parameters randomly chosen as in eq. (3.2), for a subset of approximately $10^4$ points satisfying LEP constraints.[1] We can see that small values of $\xi$ are associated with a very light mass for the $h_1$ state. Such light Higgs states are not excluded by current experimental bounds as long as their reduced coupling to the gauge bosons is small enough. Indeed, the LEP exclusion limits on Higgs boson production gives an upper bound on the reduced coupling of a Higgs boson to the gauge bosons as a function of its mass. The reduced coupling $R_i$ is defined as the coupling $ZZh_i$, divided by the corresponding Standard Model coupling:

$$R_i = h_{i1}\cos\beta + h_{i2}\sin\beta\,, \tag{3.3}$$

where $h_{i1}, h_{i2}$ are the $S_1, S_2$ components of the Higgs state $h_i$, respectively. One has

[1]Larger subsets would only increase the density of points on figure 1, not its global aspect.

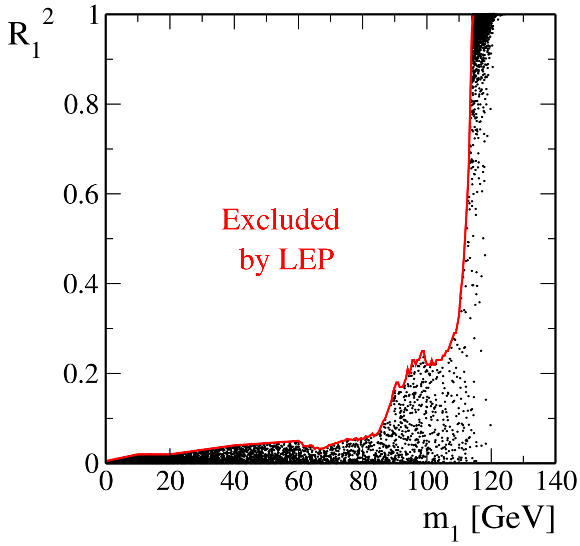


**Figure 2:** Reduced coupling of the lightest Higgs state to the SM gauge bosons as a function of its mass for the same points as in figure 1. The solid line indicates the LEP exclusion limit [14].

$0 < |R_1| < 1$ and unitarity implies

$$\sum_{i=1}^{5} R_i^2 = 1\,. \tag{3.4}$$

In figure 2 we plot the square of the reduced coupling of the lightest Higgs state, $R_1^2$, versus $m_1$ for the same set of points as in figure 1. We also display the LEP exclusion curve [14], from which one can see that the light $h_1$ states are indeed not excluded. The presence of a light Higgs state with a small reduced coupling to the SM gauge bosons might prove difficult to detect at future colliders. However, it is worth stressing that in our results we always have at least one light *visible* Higgs state with a large reduced coupling to the gauge bosons ($R_i^2 \gtrsim 0.5$) and a mass in the interval $112\,\mathrm{GeV} \lesssim m_i \lesssim 150\,\mathrm{GeV}$. The lower bound is fixed by the LEP limit as shown in figure 2. The upper bound can be understood from the following relation

$$\sum_{i=1}^{5} R_i^2 m_i^2 = \cos^2\beta M_{11}^2 + 2\cos\beta\sin\beta M_{12}^2 + \sin^2\beta M_{22}^2\,, \tag{3.5}$$

where $M_{11}^2, M_{12}^2, M_{22}^2$ are the Higgs boson mass matrix elements as given in the previous section. The right hand side of eq. (3.5) gives the usual NMSSM upper bound on the lightest Higgs boson mass [34], with the only difference being that the stop mixing parameter $X_t$ now depends on the CP violating phase $\theta$, as seen in eq. (2.13).

Assuming $m_t^{pole} = 175\,\mathrm{GeV}$, $M_{SUSY} = 350\,\mathrm{GeV}$ and maximal stop mixing, one obtains $m_1 \lesssim 125\,\mathrm{GeV}$ with this upper bound being saturated for small $\tan\beta$, namely $\tan\beta \simeq 2.7$ [34]. One can check from figures 1 and 2 that this upper bound is only reached by a few points in our set. The dense band of points in figure 1 corresponds to cases where $h_1$ is a SM-like Higgs state with $R_1 \simeq 1$. In this case, the lower bound from LEP is $m_1 \gtrsim 114\,\mathrm{GeV}$ and the upper bound, from eq. (3.5), is $m_1 \lesssim 125\,\mathrm{GeV}$ as explained above.



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We have also applied the exclusion limit from LEP on Higgs bosons associated production ($e^+e^- \to hA$ in the MSSM). This provides an upper bound on the $Zh_ih_j$ reduced coupling as a function of $m_i + m_j$ for $m_i \simeq m_j$ [15]. The $Zh_ih_j$ reduced coupling is the equivalent of $\cos(\beta - \alpha)$ in the MSSM and is here defined as

$$R'_{ij} = h_{i4}(h_{j2}\cos\beta - h_{j1}\sin\beta) - (i \leftrightarrow j)\,, \tag{3.6}$$

with $h_{i4}$ the CP-odd doublet $P$ component of $h_i$. We have also taken into account the LEP limit on the charged Higgs boson mass [16], which here simply reads $m_{H^\pm} > 89.6\,\text{GeV}$. By inspection of eq. (2.24), this limit translates into having opposite signs for the phases $\theta$ and $\delta$.

As we will see in the next section, charginos play an important role in the computation of $\varepsilon_K$. The tree level chargino mass matrix in the $(\tilde{W}, \tilde{H})$ basis reads

$$\mathcal{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta e^{-i\theta} \\ \sqrt{2}M_W\cos\beta & -\lambda x \end{pmatrix}, \tag{3.7}$$

where $M_2$ is the soft wino mass. We randomly scanned in the following interval for $M_2$

$$100\,\text{GeV} < M_2 < 250\,\text{GeV} \tag{3.8}$$

and applied the LEP bound on the chargino mass, $m_{\tilde{\chi}^\pm_{1,2}} > 103.5\,\text{GeV}$ [35]. Finally, we checked that for each point the stop masses also satisfied the associated LEP limit $m_{\tilde{t}_{1,2}} > 100\,\text{GeV}$ [36].

## 4. Indirect CP violation in $K^0 - \bar{K}^0$ mixing

In the framework of the NMSSM with SCPV, all the SUSY parameters are real. Even so, the physical phases of the Higgs doublets and singlet appear in the scalar fermion, chargino and neutralino mass matrices, as well as in several interaction vertices. In this section we will explore whether or not these physical phases can account for the experimental value of $\varepsilon_K$, $\varepsilon_K = (2.271 \pm 0.017) \times 10^{-3}$ [37].

It is worth stressing that in this scenario the SM does not provide any contribution to the CP violating observables, since the CKM matrix is real. This can be clarified by noting that since $N$ is a singlet field, it does not couple to the quarks, and although both Higgs doublets do couple, the phase associated with these couplings can be rotated away by means of a redefinition of the right-handed quark fields. Since charged currents are purely left-handed, these phases do not show up in the $V_{\text{CKM}}$, which is thus real.

Let us now proceed to compute the contributions to the indirect CP violation parameter of the kaon sector, which is defined as

$$\varepsilon_K \simeq \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\text{Im}\mathcal{M}_{12}}{\Delta m_K}\,. \tag{4.1}$$

In the latter $\Delta m_K$ is the long- and short-lived kaon mass difference, and $\mathcal{M}_{12}$ is the off-diagonal element of the neutral kaon mass matrix, which is related to the effective hamiltonian that governs $\Delta S = 2$ transitions as

$$\mathcal{M}_{12} = \frac{\langle K^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | \bar{K}^0 \rangle}{2 m_K}\,, \qquad \text{with} \qquad \mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_i c_i \mathcal{O}_i\,. \tag{4.2}$$

Here $c_i$ are the Wilson coefficients and $\mathcal{O}_i$ the local operators. In the presence of SUSY contributions, the Wilson coefficients can be decomposed as $c_i = c_i^W + c_i^{H^\pm} + c_i^{\tilde{\chi}^\pm} + c_i^{\tilde{g}} + c_i^{\tilde{\chi}^0}$. As discussed in [24], in the present class of models where there are no contributions from the SM, the chargino mediated box diagrams give the leading supersymmetric contribution, and the $\Delta S = 2$ transition is largely dominated by the $(V-A)$ four fermion operator $\mathcal{O}_1$. Working in the weak basis for the $\tilde{W} - \tilde{H}$, rather than in the physical chargino basis, and using the mass insertion approximation for the internal squarks, we have verified that the $\varepsilon_K$ receives the leading contribution from the box diagrams depicted in figure 3. In the limit of degenerate masses for the left-handed up-squarks, $\mathrm{Im}\mathcal{M}_{12}$ is given by [24]

$$\begin{aligned}\mathrm{Im}\mathcal{M}_{12} = \; & \frac{2 G_F^2 f_K^2 m_K m_W^4}{3\pi^2 \langle m_{\tilde{q}} \rangle^8} (V_{td}^* V_{ts}) m_t^2 \left| e^{i\theta}\, m_{\tilde{W}} - \cot\beta\, m_{\tilde{H}} \right| \times \\ & \times \left\{ \Delta A_U \sin[\varphi_\chi - \theta]\, \left(M_{\tilde{Q}}^2\right)_{12} I(r_{\tilde{W}}, r_{\tilde{H}}, r_{\tilde{u}_L}, r_{\tilde{t}_R}) \right\}. \end{aligned} \tag{4.3}$$

In the above equation, $f_K$ is the kaon decay constant and $m_K$ the kaon mass [37]; $V_{ij}$ are the $V_{\rm CKM}$ elements, whose numerical values ($V_{td} = 0.0066$ and $V_{ts} = -0.04$) reflect the fact that we are dealing with a flat unitarity triangle; $\langle m_{\tilde{q}} \rangle$ is the average squark mass, which we take equal to $M_{\rm SUSY}$; $m_{\tilde{W}} = M_2$ is the wino mass, $m_{\tilde{H}} = \mu$ is the higgsino mass and $\varphi_\chi = \arg(e^{i\theta} m_{\tilde{W}} - \cot\beta\, m_{\tilde{H}})$.[2] The non-universality in the $LL$ soft breaking masses is parameterized by $\left(M_{\tilde{Q}}^2\right)_{12}$, which we choose taking into account the bounds from the analysis in ref. [38]. As discussed in ref. [24], a sizable non-universality in the soft trilinear terms is crucial, and is here parameterized by $\Delta A_U \equiv A_U^{13} - A_U^{23}$. In the following, we assume $\Delta A_U = 500\,\mathrm{GeV}$. Finally, $I$ is the loop function, with $r_i = m_i^2 / \langle m_{\tilde{q}} \rangle^2$ [24].

By scanning the parameter space of the model, we have verified that one can find sets of parameters that satisfy the minimization conditions of the Higgs potential, have an associated Higgs boson spectrum compatible with LEP searches and still succeed in generating the observed value of $\varepsilon_K$. From eq. (4.3), it appears that $\varepsilon_K$ depends on $1/M_{\rm SUSY}^8$. It is therefore difficult to saturate the experimental value of $\varepsilon_K$ for large values of $M_{\rm SUSY}$. On the other hand, too small values of $M_{\rm SUSY}$ might generate a light Higgs boson spectrum, already excluded by LEP. In order to accommodate both constraints, we took $M_{\rm SUSY} = 350\,\mathrm{GeV}$, as already referred to in section 3. All other parameters are as in eqs. (3.2, 3.8) and we assumed $m_t^{\rm pole} = 175\,\mathrm{GeV}$ and maximal stop mixing, as in the previous section.

In figure 4, we plot the possible values of $\varepsilon_K$ as a function of $\tan\beta$. The maximal values of $\varepsilon_K$ are obtained for the low $\tan\beta$ regime, and the experimental bounds on $\varepsilon_K$ require $\tan\beta \lesssim 3.8$. Recall that the lower bound for $\tan\beta$ is consistent with the analysis conducted in section 3. Apart from the explicit dependence of eq. (4.3) on $\tan\beta$, one should bear in

[2]Our conventions differ from ref. [24] in that $\mu_{eff} \to -\mu$.

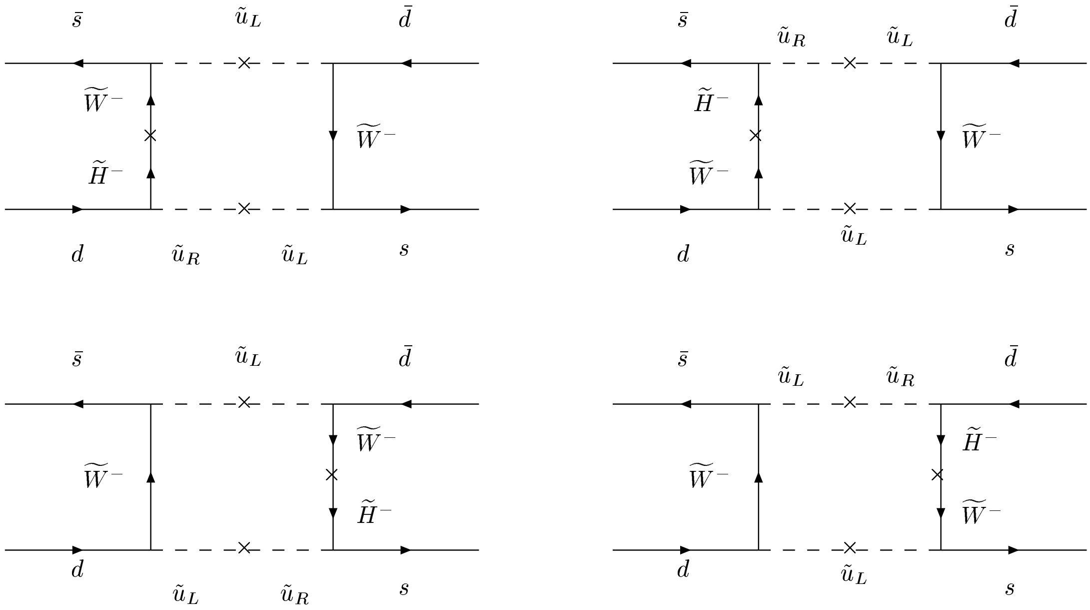


**Figure 3:** Box diagrams associated with the leading chargino contribution to $\varepsilon_K$.

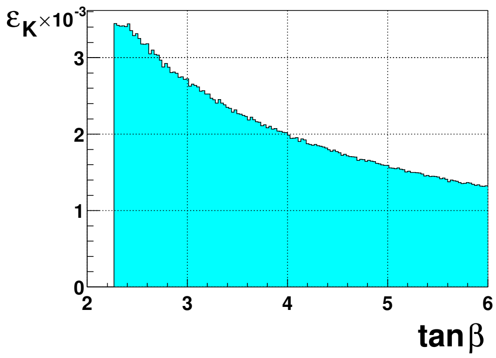


**Figure 4:** Values of $\varepsilon_K$ as function of $\tan\beta$ (shaded area) for $m_t^{\rm pole} = 175\,\text{GeV}$, $M_{\rm SUSY} = 350\,\text{GeV}$ and maximal stop mixing. The other parameters are randomly chosen as in eqs. (3.2) and (3.8).

mind that there are also implicit dependences associated with the other parameters as well as with the various experimental bounds imposed on the mass spectrum. It is therefore difficult to reproduce analytically the observed upper bound on $\varepsilon_K$ as a function of $\tan\beta$.

In figure 5, we plot the maximal value of $\varepsilon_K$ in distinct regions of the $\lambda - \kappa$ plane. The remaining parameters are chosen in order to maximize $\varepsilon_K$ and still comply with the experimental bounds. As one can see from this figure, having $\varepsilon_K \sim 2 \times 10^{-3}$ is associated with values of $\kappa$ and $\lambda$ in the range $[0, 0.6]$. In other words, one can easily saturate $\varepsilon_K$ in a vast region of the parameter space.

As expected from the inspection of eq. (4.3), there is a strong dependence of $\varepsilon_K$ on the phases associated with the Higgs fields. In figure 6, we display contour plots for the maximal values of $\varepsilon_K$ in the plane generated by the phases $\theta$ and $\delta$. Although, *a priori*,

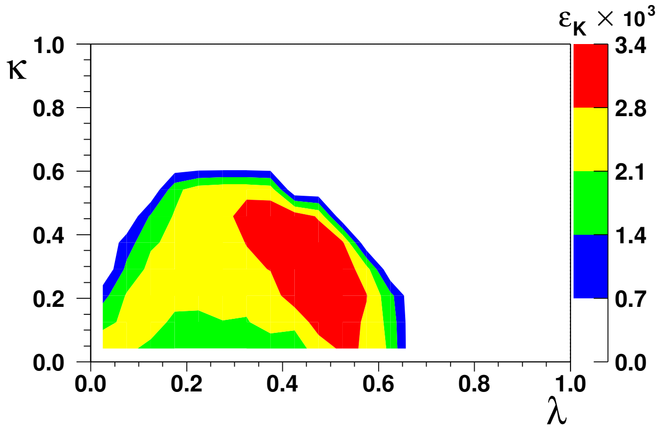


**Figure 5:** Contours for the maximum value of $\varepsilon_K$ in the $\lambda$–$\kappa$ plane for $m_t^{\rm pole} = 175\,{\rm GeV}$, $M_{\rm SUSY} = 350\,{\rm GeV}$, maximal stop mixing and the other parameters as in eqs. (3.2) and (3.8).

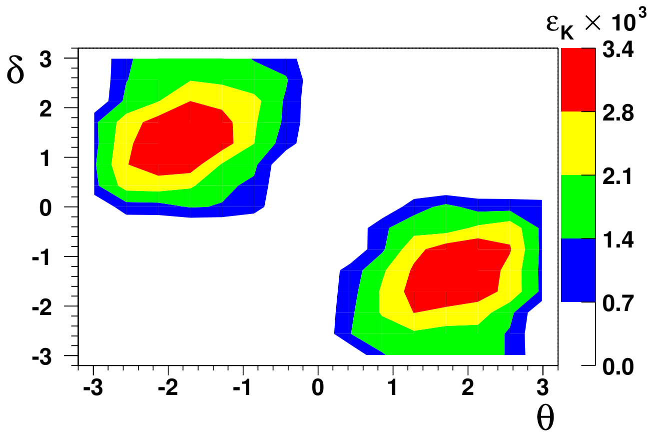


**Figure 6:** Contours for the maximum value of $\varepsilon_K$ in the $\theta$–$\delta$ plane. All parameters as in figure 5.

all the values for the phases $\theta$ and $\delta$ in $[-\pi, \pi]$ are allowed, it is clear from figure 6 that the saturation of the experimental value of $\varepsilon_K$ can only be achieved for significant values of the singlet and doublet phases. This feature becomes more evident in figure 7, where we show the values of $\varepsilon_K$ as a function of the singlet phase, $\varphi_3 = \delta/3$. The saturation of the experimental value of $\varepsilon_K$ requires the singlet phase to be $|\varphi_3| \gtrsim 0.15$. We will discuss in the conclusions the implications of such large CP violating phases.

## 5. Conclusions and discussion

We have shown that in the framework of the NMSSM with an extra tadpole term for the singlet in the scalar potential, it is possible to have spontaneous CP violation while complying with the LEP constraints on the Higgs boson mass. Although in this scenario $V_{\rm CKM}$ is real, the experimental value of $\varepsilon_K$ can be saturated in large parts of the parameter space of the model, without requiring any fine tuning. This may be of special interest for

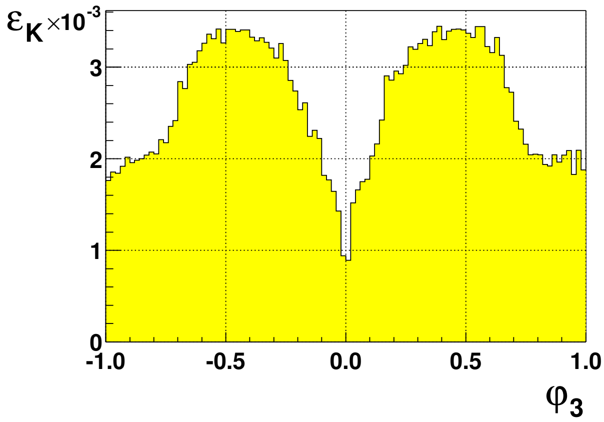


**Figure 7:** Values of $\varepsilon_K$ as function of the singlet phase $\varphi_3 = \delta/3$ (grey area). All parameters as in figure 5.

electroweak baryogenesis, as it has been shown that a strong first order electroweak phase transition is possible in the NMSSM, as opposed to the MSSM, due to the additional trilinear Higgs boson couplings in the lagrangian [39].

The presence of the extra tadpole term $\xi$ is crucial to conciliate spontaneous CP violation with the LEP constraints on the Higgs boson mass. Moreover, the $\xi$ term also solves the domain wall problem associated with the spontaneous breaking of the $\mathbb{Z}_3$ symmetry, which is present if $\xi = 0$. On the other hand, since CP is spontaneously broken at the electroweak scale, CP domain walls may appear, which are cosmologically excluded. An elegant solution to this problem, along the lines of the $\mathbb{Z}_3$ domain walls solution, is to assume that gravitational interactions also explicitly break CP. In the low energy scalar potential, one then obtains an explicit CP violating tadpole term, i.e. a complex $\xi$ term. A phase in the $\xi$ term would not change the results derived here, since its only effect is to generate a shift in the singlet phase $\delta$.

Regarding the large phase regime favoured by the saturation of $\varepsilon_K$, one should recall that $\theta$ and $\delta$ are flavour-conserving phases, and might generate sizable contributions to the electron, neutron and mercury atom EDMs, which can represent a serious challenge to the survival of the model.

Although we postpone a detailed evaluation of the EDM problem inherent to this specific model to a forthcoming work [42], it is important to comment here on some possible ways to overcome the EDM constraints, and how the latter can be applied to this particular case. We first notice that in the presence of a small singlet coupling $\lambda$, as allowed in our results (see figure 5), the EDM constraints on $\delta$ become less stringent [40]. Nevertheless, one still has to suppress the effects of the large $\theta$ phase, and in principle this can be achieved in the following ways: reinforcing the non-universality of the trilinear terms (i.e. requiring the diagonal terms to be much smaller than the off-diagonal ones or having matrix-factorizable $A$ terms), the existence of cancellations between the several SUSY contributions, and the suppression of the EDMs by a heavy SUSY spectrum [6]. Although



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the first possibility might require some model building, it is surely one of the most efficient ways to suppress EDM contributions stemming from the sfermion soft terms. The cancellation hypothesis is not a very attractive solution, given that in general it requires some fine-tuning, and it is model-dependent. However, one should point here that the analysis of [6] does not necessarily apply to the present model. In this case, we have a vanishing gluino phase, and the soft breaking terms are assumed to be free at the weak scale. In addition, one cannot establish a direct connection between the free parameters of our model, and those of the models analysed in [6], namely the independent phases. Therefore, it is possible that one can find regions of the parameter space where cancellations do indeed occur [42]. Finally, the third solution is harder to implement, since one requires low values of $M_{SUSY}$ in order to saturate the experimental value of $\varepsilon_K$. Nevertheless, a heavy gluino and photino might also provide a significant suppression of the EDMs, although these may give rise to some cosmological problems, among others. We conclude the EDM discussion by noticing that in view of the considerably large parameter space allowed in our results, none of the above possibilities should be completely disregarded.

Concerning the other CP-violating observables, namely $\varepsilon'/\varepsilon$ and the CP asymmetry of the $B_d$ meson decay ($a_{J/\psi K_S}$), it has been pointed out that this class of models can generate sizable contributions, although saturating the experimental values generally favours a regime of large phases and maximal $LR$ squark mixing [22, 41]. A complete analysis of these issues has yet to be done.

In this work, we pointed out that the NMSSM with an extra tadpole term in the scalar potential appears to be an excellent candidate for a scenario of SCPV. Having a $\mathbb{Z}_3$ invariant superpotential preserves the original motivation of the NMSSM to solve the $\mu$ problem of supersymmetry. The extra tadpole term for the singlet cures the domain wall problem of the $\mathbb{Z}_3$ invariant model and allows the spontaneous breaking of CP. In this model, one can simultaneously saturate $\varepsilon_K$ and obtain a particle spectrum compatible with the current experimental bounds. The analysis of the EDM problem for this class of non-minimal SUSY models with SCPV is surely highly desirable and critical, and might prove to be a major test to the viability of these scenarios.

The presently available experimental data on CP violation does not allow to distinguish whether the CP symmetry is spontaneously or explicitly broken. Even so, with the advent of the LHC and eventually of the Linear Collider, direct searches of the Higgs bosons, and the measurement the associated couplings to the SM gauge bosons, should allow to disentangle this model from the MSSM.

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## References


[1] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Evidence for the $2\pi$ decay of the $K_2^0$ meson*, *Phys. Rev. Lett.* **13** (1964) 138.

[2] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531;
M. Kobayashi and T. Maskawa, *CP-violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.

[3] M.B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. Quimbay, *Standard model CP-violation and baryon asymmetry. Part 2. Finite temperature*, *Nucl. Phys.* **B 430** (1994) 382 [hep-ph/9406289].

[4] G. 't Hooft et al. eds., *Recent developments in gauge theories*, in *Proceedings of the Nato Advanced Summer Institute*, Cargèse, 1979, Plenum, New York 1980.

[5] S. Khalil, T. Kobayashi and A. Masiero, *CP-violation in supersymmetric models with nondegenerate A-terms*, *Phys. Rev.* **D 60** (1999) 075003 [hep-ph/9903544];
S. Khalil and T. Kobayashi, *Supersymmetric CP-violation $\epsilon'/\epsilon$ due to asymmetric A-matrix*, *Phys. Lett.* **B 460** (1999) 341 [hep-ph/9906374];
G.C. Branco, M.E. Gomez, S. Khalil and A.M. Teixeira, *Supersymmetry and a rationale for small CP violating phases*, *Nucl. Phys.* **B 659** (2003) 119.

[6] S. Abel, S. Khalil and O. Lebedev, *EDM constraints in supersymmetric theories*, *Nucl. Phys.* **B 606** (2001) 151 [hep-ph/0103320];
S. Khalil, *CP-violation in supersymmetric theories*, *Int. J. Mod. Phys.* **A 18** (2003) 1697 [hep-ph/0212050].

[7] T.D. Lee, *A theory of spontaneous T violation*, *Phys. Rev.* **D 8** (1973) 1226;
S. Weinberg, *Gauge theory of CP-violation*, *Phys. Rev. Lett.* **37** (1976) 657.

[8] R.N. Mohapatra and G. Senjanovic, *Natural suppression of strong P and T noninvariance*, *Phys. Lett.* **B 79** (1978) 283;
H. Georgi, *A model for soft CP violation*, *Hadronic Journal* 1 (1978) 155;
S.M. Barr and P. Langacker, *A superweak gauge theory of CP-violation*, *Phys. Rev. Lett.* **42** (1979) 1654;
A.E. Nelson, *Naturally weak CP-violation*, *Phys. Lett.* **B 136** (1984) 387;
S.M. Barr, *Solving the strong CP problem without the Peccei-Quinn symmetry*, *Phys. Rev. Lett.* **53** (1984) 329.

[9] A. Strominger and E. Witten, *New manifolds for superstring compactification*, *Commun. Math. Phys.* **101** (1985) 341.

[10] For a review, see J.F. Gunion, G.L. Kane, H.E. Haber and S. Dawson, *The Higgs hunter's guide*, Addison Wesley 1990.

[11] N. Maekawa, *'Spontaneous' CP-violation in minimal supersymmetric standard model*, *Phys. Lett.* **B 282** (1992) 387.

[12] H. Georgi and A. Pais, *CP-violation as a quantum effect*, *Phys. Rev.* **D 10** (1974) 1246.

[13] A. Pomarol, *Higgs sector CP-violation in the minimal supersymmetric model*, *Phys. Lett.* **B 287** (1992) 331 [hep-ph/9205247].

[14] LEP Higgs Working Group, *Search for the standard model Higgs boson at LEP*, Note 2002/01.

[15] LEP Higgs Working Group, *Searches for the neutral Higgs bosons of the MSSM at LEP*, Note 2001/04.

[16] LEP Higgs working group, *Search for charged Higgs bosons at LEP*, Note 2001/05.

[17] H.P. Nilles, M. Srednicki and D. Wyler, *Weak interaction breakdown induced by supergravity*, *Phys. Lett.* **B 120** (1983) 346.

[18] J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev.* **D 39** (1989) 844.

[19] J.C. Romão, *Spontaneous CP-violation in SUSY models: a no-go theorem*, *Phys. Lett.* **B 173** (1986) 309.

[20] K.S. Babu and S.M. Barr, *Spontaneous CP-violation in the supersymmetric Higgs sector*, *Phys. Rev.* **D 49** (1994) 2156 [hep-ph/9308217].

[21] N. Haba, M. Matsuda and M. Tanimoto, *Spontaneous CP-violation and Higgs masses in the next-to-minimal supersymmetric model*, *Phys. Rev.* **D 54** (1996) 6928 [hep-ph/9512421].

[22] A. Pomarol, *Spontaneous CP-violation in supersymmetric theories*, *Phys. Rev.* **D 47** (1993) 273 [hep-ph/9208205].

[23] A.T. Davies, C.D. Froggatt and A. Usai, *Light Higgs boson in the spontaneously CP-violating NMSSM*, *Phys. Lett.* **B 517** (2001) 375 [hep-ph/0105266].

[24] G.C. Branco, F. Krüger, J.C. Romão and A.M. Teixeira, *Spontaneous CP-violation in the next-to-minimal supersymmetric standard model revisited*, *J. High Energy Phys.* **07** (2001) 027 [hep-ph/0012318].

[25] U. Ellwanger, *Nonrenormalizable interactions from supergravity, quantum corrections and effective low-energy theories*, *Phys. Lett.* **B 133** (1983) 187.

[26] S.A. Abel, *Destabilising divergences in the NMSSM*, *Nucl. Phys.* **B 480** (1996) 55 [hep-ph/9609323].

[27] S.A. Abel, S. Sarkar and P.L. White, *On the cosmological domain wall problem for the minimally extended supersymmetric standard model*, *Nucl. Phys.* **B 454** (1995) 663 [hep-ph/9506359].

[28] C. Panagiotakopoulos and K. Tamvakis, *Stabilized NMSSM without domain walls*, *Phys. Lett.* **B 446** (1999) 224 [hep-ph/9809475].

[29] C. Panagiotakopoulos and K. Tamvakis, *New minimal extension of MSSM*, *Phys. Lett.* **B 469** (1999) 145 [hep-ph/9908351];
A. Dedes, C. Hugonie, S. Moretti and K. Tamvakis, *Phenomenology of a new minimal supersymmetric extension of the standard model*, *Phys. Rev.* **D 63** (2001) 055009 [hep-ph/0009125];
C. Panagiotakopoulos and A. Pilaftsis, *Higgs scalars in the minimal non-minimal supersymmetric standard model*, *Phys. Rev.* **D 63** (2001) 055003 [hep-ph/0008268]; *Light charged Higgs boson and supersymmetry*, *Phys. Lett.* **B 505** (2001) 184 [hep-ph/0101266].

[30] S.W. Ham, S.K. Oh and H.S. Song, *Spontaneous violation of the CP symmetry in the Higgs sector of the next-to-minimal supersymmetric model*, *Phys. Rev.* **D 61** (2000) 055010 [hep-ph/9910461].



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[31] Y. Okada, M. Yamaguchi and T. Yanagida, *Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model*, *Prog. Theor. Phys.* **85** (1991) 1; *Renormalization group analysis on the Higgs mass in the softly broken supersymmetric standard model*, *Phys. Lett.* **B 262** (1991) 54;
A. Yamada, *Radiative corrections to the Higgs masses in the minimal supersymmetric standard model*, *Phys. Lett.* **B 263** (1991) 233;
J.R. Ellis, G. Ridolfi and F. Zwirner, *Radiative corrections to the masses of supersymmetric Higgs bosons*, *Phys. Lett.* **B 257** (1991) 83; *On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches*, *Phys. Lett.* **B 262** (1991) 477;
A. Brignole, J. Ellis, G. Ridolfi, F. Zwirner, *The supersymmetric charged Higgs boson mass and LEP phenomenology*, *Phys. Lett.* **B 271** (1991) 123;
R. Barbieri, M. Frigeni and F. Caravaglios, *The supersymmetric Higgs for heavy superpartners*, *Phys. Lett.* **B 258** (1991) 167;
H.E. Haber and R. Hempfling, *Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than $m_Z$?*, *Phys. Rev. Lett.* **66** (1991) 1815.

[32] U. Ellwanger, *Radiative corrections to the neutral Higgs spectrum in supersymmetry with a gauge singlet*, *Phys. Lett.* **B 303** (1993) 271 [hep-ph/9302224];
T. Elliott, S.F. King and P.L. White, *Supersymmetric Higgs bosons at the limit*, *Phys. Lett.* **B 305** (1993) 71 [hep-ph/9302202]; *Squark contributions to Higgs boson masses in the next-to-minimal supersymmetric standard model*, *Phys. Lett.* **B 314** (1993) 56 [hep-ph/9305282];
*Radiative corrections to Higgs boson masses in the next-to-minimal supersymmetric standard model*, *Phys. Rev.* **D 49** (1994) 2435 [hep-ph/9308309].

[33] M. Carena, H.E. Haber, S. Heinemeyer, W. Hollik, C.E.M. Wagner, G. Weiglein, *Reconciling the two-loop diagrammatic and effective field theory computations of the mass of the lightest CP-even Higgs boson in the MSSM*, *Nucl. Phys.* **B 580** (2000) 29 [hep-ph/0001002];
J.R. Espinosa and R.-J. Zhang, *Complete two-loop dominant corrections to the mass of the lightest CP-even Higgs boson in the minimal supersymmetric standard model*, *Nucl. Phys.* **B 586** (2000) 3 [hep-ph/0003246];
A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *On the $O(\alpha_t^2)$ two-loop corrections to the neutral Higgs boson masses in the MSSM*, *Nucl. Phys.* **B 631** (2002) 195 [hep-ph/0112177].

[34] U. Ellwanger and C. Hugonie, *Masses and couplings of the lightest Higgs bosons in the $(M+1)$SSM*, *Eur. Phys. J.* **C 25** (2002) 297 [hep-ph/9909260].

[35] LEP SUSY Working Group, *Combined LEP chargino results*, Note 01-03.1.

[36] LEP SUSY Working Group, *Combined LEP stop and sbottom results*, Note 02-02.1.

[37] Particle Data Group collaboration, K. Hagiwara et al., *Review of particle physics*, *Phys. Rev.* **D 66** (2002) 010001.

[38] M. Ciuchini et al., *$\Delta M_K$ and $\epsilon_K$ in SUSY at the next-to-leading order*, *J. High Energy Phys.* **10** (1998) 008 [hep-ph/9808328].

[39] M. Pietroni, *The electroweak phase transition in a nonminimal supersymmetric model*, *Nucl. Phys.* **B 402** (1993) 27 [hep-ph/9207227];
A.T. Davies, C.D. Froggatt and R.G. Moorhouse, *Electroweak baryogenesis in the next to minimal supersymmetric model*, *Phys. Lett.* **B 372** (1996) 88 [hep-ph/9603388];
S.J. Huber and M.G. Schmidt, *SUSY variants of the electroweak phase transition*, *Eur. Phys. J.* **C 10** (1999) 473 [hep-ph/9809506];

M. Bastero-Gil, C. Hugonie, S.F. King, D.P. Roy and S. Vempati, *Does LEP prefer the NMSSM?*, *Phys. Lett.* **B 489** (2000) 359 [hep-ph/0006198].

[40] M. Matsuda and M. Tanimoto, *Explicit CP-violation of the Higgs sector in the next-to-minimal supersymmetric Standard Model*, *Phys. Rev.* **D 52** (1995) 3100 [hep-ph/9504260].

[41] O. Lebedev, *Constraining SUSY models with spontaneous CP-violation via $B \to \psi K_S$*, *Int. J. Mod. Phys.* **A 15** (2000) 2987 [hep-ph/9905216].

[42] C. Hugonie, J.C. Romão and A.M. Teixeira, work in progress.